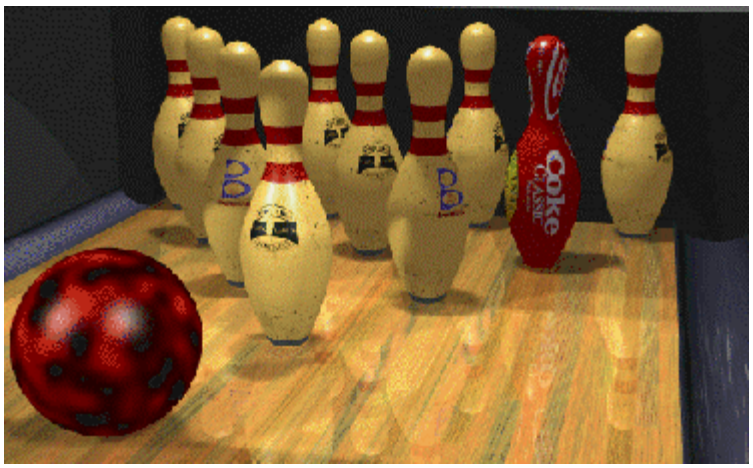
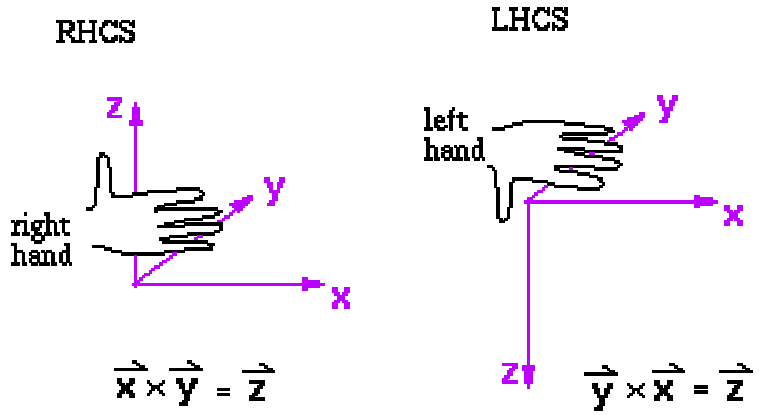
CAC PHEP BIEN ÑOI 3 CHIEU



Dan nhap

- Cung mot loai ñoi töông co the xuat hien trong nhieu canh va xuat hien nhieu lan trong mot canh vôi cac phöông vò, mau sac khac nhau.
- Neu ta co cac mo hình ñoi töông tot, ta co the phat sinh ra cac ñoi töông khac nhau tö mot mo hình duy nhat nhô cac phep bien ñoi.
- Cac phep bien ñoi quan trong nhat la cac phep bien ñoi Affine va cac phep chieu.

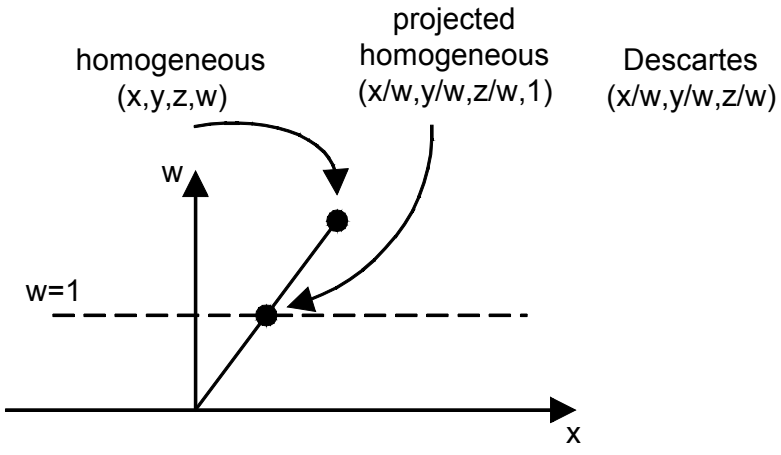
He toa ño ban tay phai/ban tay trai



- He toa ño theo quy öôc ban tay phai: ñe ban tay phai sao cho ngon cai höông theo truc z, khi nam tay lai, cac ngon tay chuyen ñong theo höông tö truc x ñen truc y.
- He toa ño theo quy öôc ban tay trai: ñe ban tay phai sao cho ngon cai höông theotruc z, khi nam tay lai, cac ngon tay chuyen ñong theo höông tö truc x ñen truc y.

He toa ño thuan nhat (Homogeneous Coordinates)

- Moi ñiem (x, y, z) trong khong gian Descartes ñöôc bieu dien bôi mot bo bon toa ño trong khong gian 4 chieu thu gon (hx, hy, hz, h). Ngöôi ta thöông chon h=1.



- $(x, y, z)_{Descartes}$ $(x, y, z, 1)_{Homogeneous}$
- $(x, y, z, w)_{Homogeneous}$ $(x/w, y/w, z/w)_{Descartes}$ $(w \neq 0)$.

Cac phep bien ñoi tuyen tính

- Phep bien ñoi tuyen tính la to hôp cua cac PBÑ:
  - Tæ le
  - Quay
  - Bien dang va
  - Ñoi xöng

$$(x'\ \ y'\ \ z') = (x\ \ y\ \ z)\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} \text{ vôi } \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} \neq 0$$

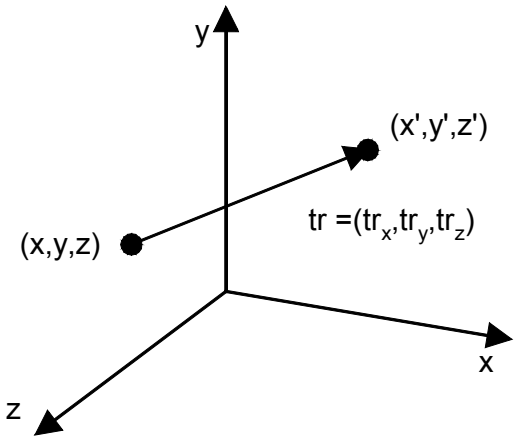
- Cac tính chat cua cac phep bien ñoi tuyen tính
  - Thoa man tính chat ve to hôp tuyen tính.

$$T(s_1P_1 + s_2P_2) = s_1T(P_1) + s_2T(P_2)$$

  - Goc toa ño la ñiem bat ñong.
  - Anh cua ñöông thang la ñöông thang.
  - Anh cua cac ñöông thang song song la cac ñöông thang song song.
  - Bao toan tæ le khoang cach
  - To hôp cac phep bien ñoi co tính phan phoi

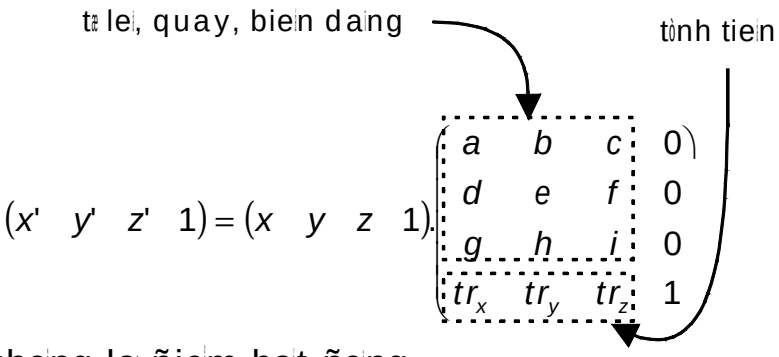
Phep tònh tien

- Dòch chuyen mot ñiem tö vò trí ñen vò trí khac trong khong gian theo vector offset **tr.**



Phep bien ñoi Affine

- Phep bien ñoi Affine la to hôp cua cac phep bien ñoi:
  - Tuyen tính
  - Tònh tien



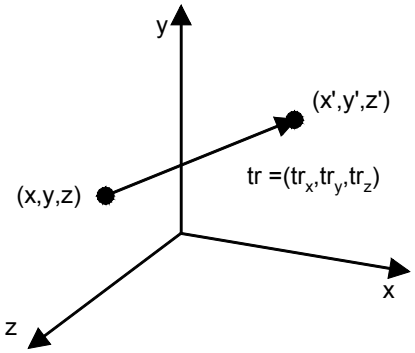
- Cac tính chat
  - Goc toa ño khong la ñiem bat ñong.
  - Anh cua ñöông thang la ñöông thang.
  - Anh cua cac ñöông thang song song la cac ñöông thang song song.
  - Bao toan tæ le khoang cach
  - To hôp cac phep bien ñoi co tính phan phoi

Cac phep bien ñoi Affine cô sô

- Phep bien ñoi Affine co the xem la to hôp cua cac phep bien ñoi cô sô:
  - Tònh tien
  - Tæ le (tam tæ le ñat tai goc toa ño)
  - Quay quanh truc x
  - Quay quanh truc y
  - Quay quanh truc z
  - Ñoi xöng qua truc x, y, z*
  - Bien dang* (tam bien dang ñat tai goc toa ño)

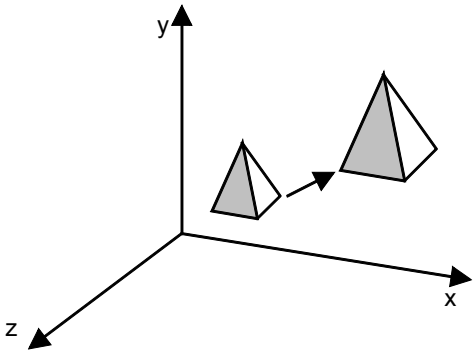
- Phep tònh tien

$$Tr(Tr_x, Tr_y, Tr_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tr_x & Tr_y & Tr_z & 1 \end{bmatrix}$$



- Phep bien ñoi tæ le

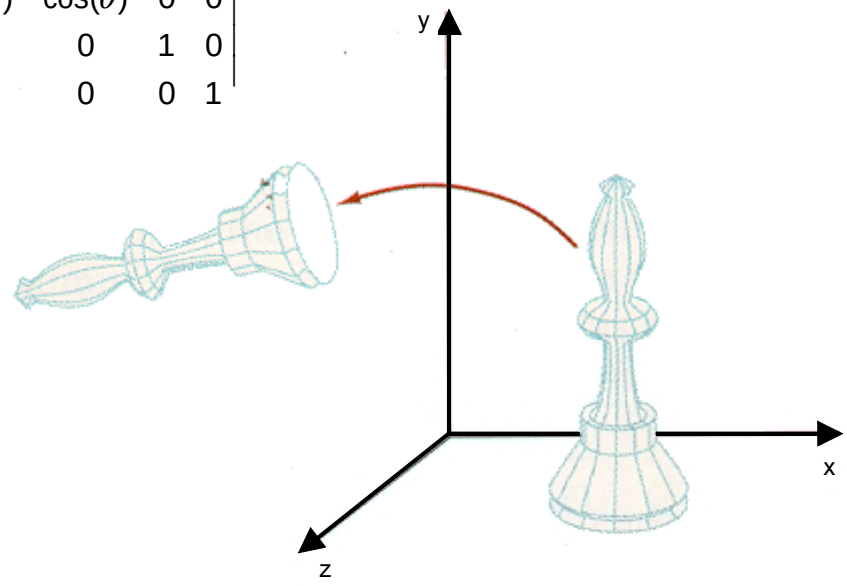
$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Khi $s_x=s_y=s_z$: ***phep ñong daïng***

- Phep quay quanh truc z

$$R(z,\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- Phep quay quanh truc x

$$R(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Phep quay quanh truc y

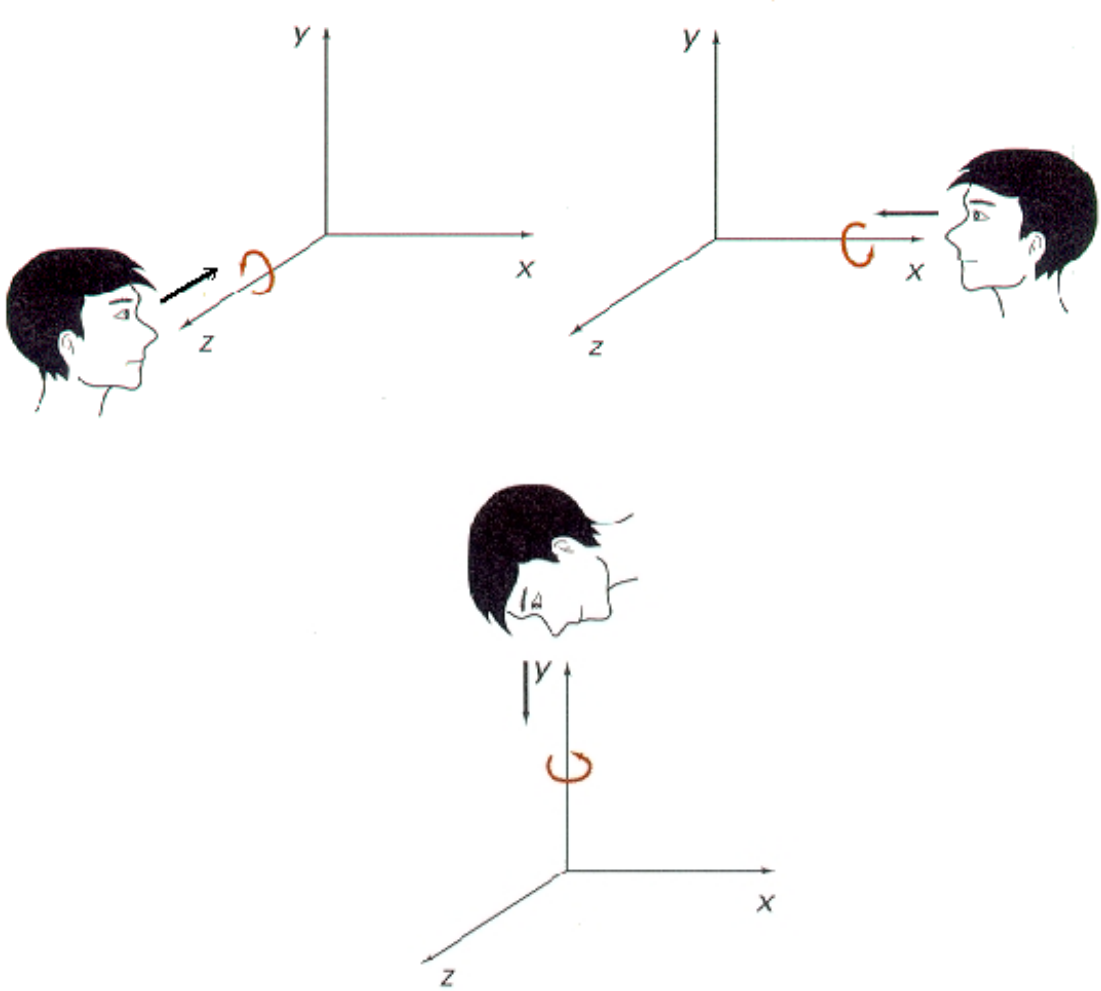
$$R(y,\theta) = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Cach xac ñònh chieu döông trong cac phep quay

  Cac ñònh nghóa ve chieu quay ñöôc dung chung cho ca he toa ño theo quy öôc ban tay phai va ban tay trai. Cu the chieu döông ñöôc ñònh nghóa nhö sau:

  - Quay quanh truc x: tö truc döông y ñen truc döông z.
  - Quay quanh truc y: tö truc döông z ñen truc döông x.
  - Quay quanh truc z: tö truc döông x ñen truc döông y.

- Ví du, xet tren he toa ño ban tay trai, khi nhìn doc tö phía truc quay ve goc toa ño, chieu döông se la chieu ngöôc chieu kim ñong ho

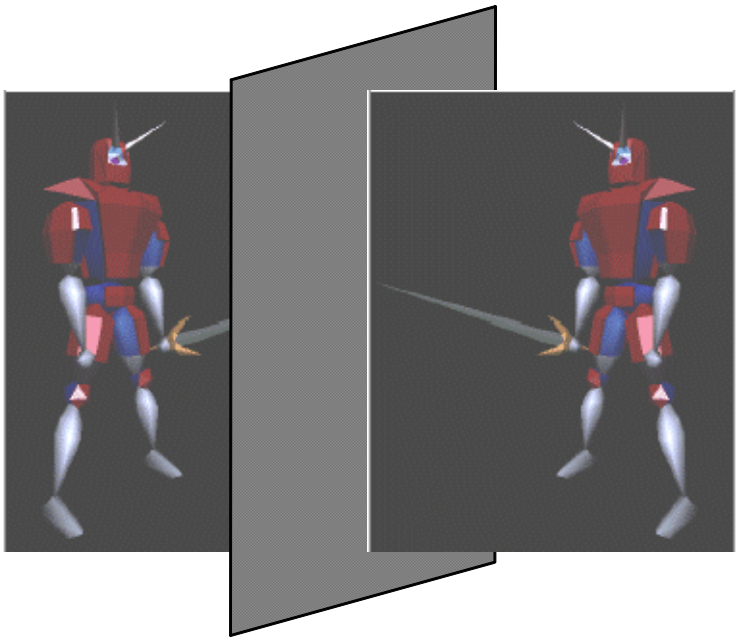


- Phep ñoi xöng qua mat phang yOz, zOx va xOy

$$Mr(x) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Mr(y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Mr(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

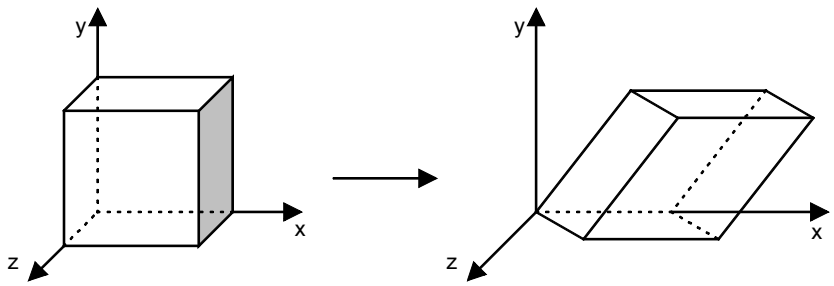


- Phep ñoi xöng qua truc x, y va z

$$M_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_z = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Phep bien dang

$$Sh = \begin{bmatrix} 1 & h_{yx} & h_{zx} & 0 \\ h_{xy} & 1 & h_{zy} & 0 \\ h_{xz} & h_{yz} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

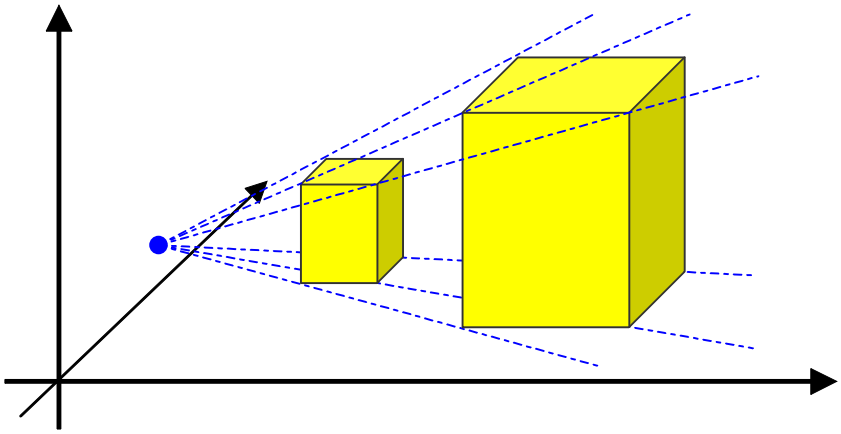


Cac phep bien ñoi Affine tong quat

- To hôp cac phep bien ñoi Affine la mot phep bien ñoi Affine.
- Moi phep bien ñoi Affine ñeu co the phan ra thanh to hôp cac phep bien ñoi Affine cô sô.

Phep tæ le vôi tam bat ky

- Phep tæ le vôi tam ñat tai ñiem (x_f, y_f, z_f) co the xet nhö to hôp cua cac phep bien ñoi cô sô:
  - Tònh tien ñiem bat ñong (x_f, y_f, z_f) ve goc toa ño.
  - Thöc hien phep bien ñoi tæ le vôi tam la goc toa ño.
  - Tònh tien ngöôc ñiem bat ñong tö goc toa ño trô ve vò trí ban ñau.

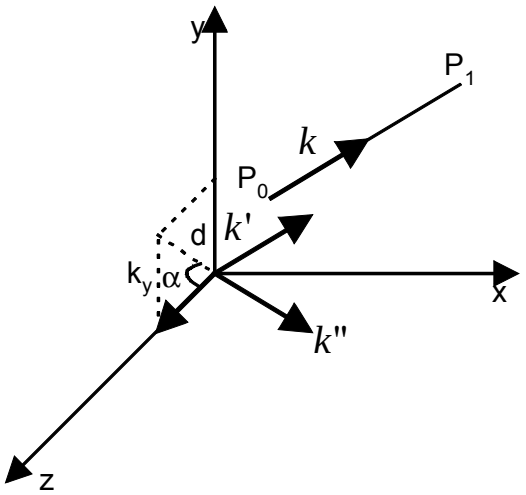


- Ma tran bien ñoi se la: $S_f(s_x,s_y,s_z) = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ (1-s_x)x_f & (1-s_y)y_f & (1-s_z)z_f & 1 \end{pmatrix}$

Phep quay quanh mot truc bat ky

- Gia sö truc quay xac ñònh bôi 2 ñiem P_1 va P_2 (chieu döông höông tö P_1 ñen P_2 the hien bôi vector **k**).

- Ap dung qui tac phan ra, ta co the bieu dien quay quanh k mot goc θ thanh day cac phep bien ñoi cô sô sau:
  - Tònh tien truc k ve goc toa ño: Tr(-P0) (thanh truc k')
  - Quay quanh truc x ñe ñat truc k' nam tren mat phang xOz: R(x,α) (thanh truc k'').
  - Goc quay ñöôc xac ñònh döa tren chieu cua k' len mat phang yOz. Ta khong can tính α cu the. Thay vao ño ta tính sin(α) va cos(α) mot cach tröc tiep.

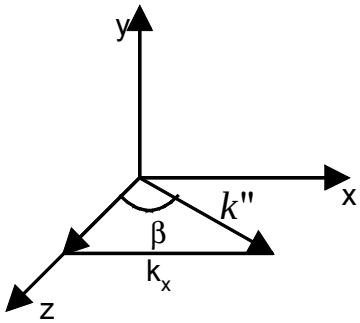


$$k = \frac{\overrightarrow{P_0P_1}}{\|P_0P_1\|},$$

$$d = \sqrt{k_y^2 + k_z^2}$$

$$\cos(\alpha) = \frac{k_z}{d}, \sin(\alpha) = \frac{k_y}{d}$$

  - Quay quanh truc y ñe ñöa truc k' ve truc z: R(y,-β). Töông tö böôc tröôc, ta khong can tính cu the β.
  - Thöc hien phep quay quanh truc z mot goc θ: R(z,θ)
  - Thöc hien chuoi cac phep bien ñoi ngöôc lai qua trình tren.



$$\cos(\beta) = \frac{d}{1} = d,$$

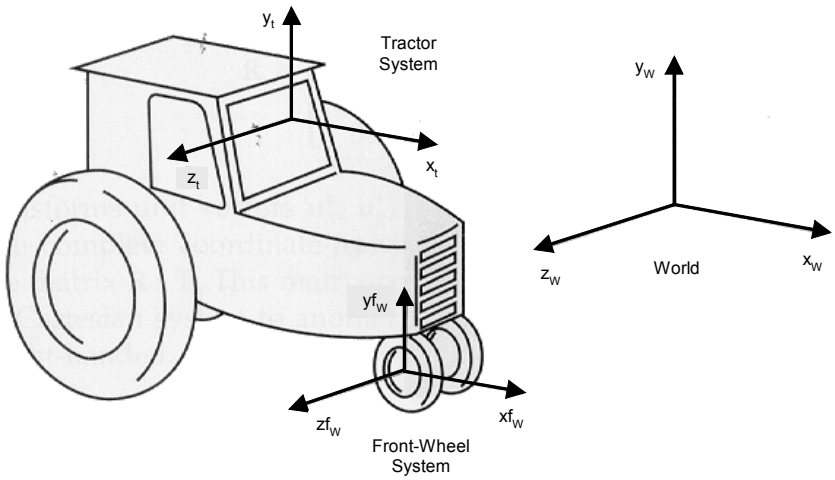
$$\sin(\beta) = \frac{k_x}{1} = k_x$$

- Nhö vay, phep quay quanh 1 truc bat ky co the ñöôc phan ra thanh chuoi cac bien ñoi cô sô sau:

$$Tr(-P_0)\ R(x,\alpha)\ R(y,-\beta)\ R(z,\theta)\ R(y,\beta)\ R(x,-\alpha)\ Tr(P_0)$$

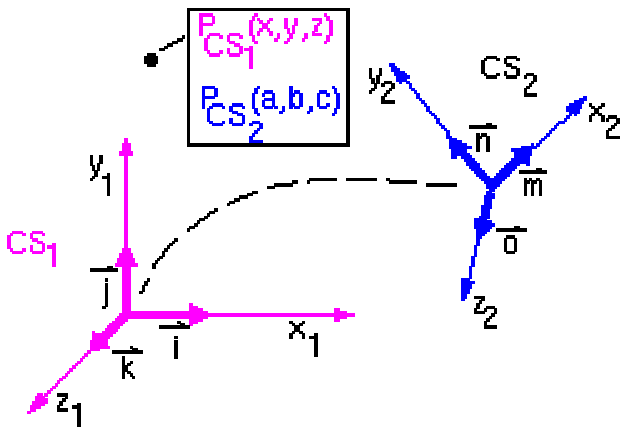
Modeling transformation

- Bien ñoi tö He toa ño ñoi töông sang He toa ño the giôi thöc.



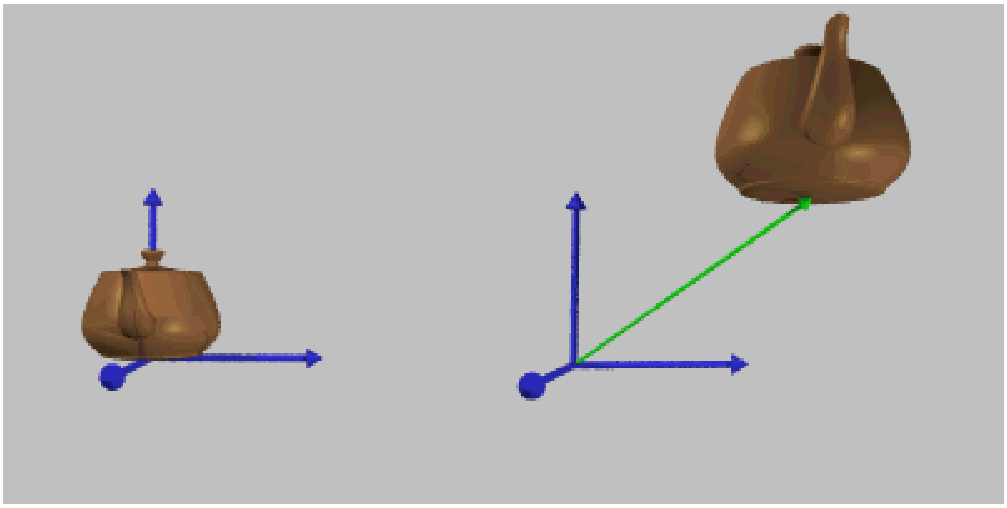
Phep bien ñoi He toa ño

- Can thöc hien mot phep quay va mot phep tònh tien (goi la Rigid boby transformation).
- Neu chuyen ñoi giöa hai he toa ño ban tay trai va ban tay phai thì can them mot phep ñoi xöng nöa.

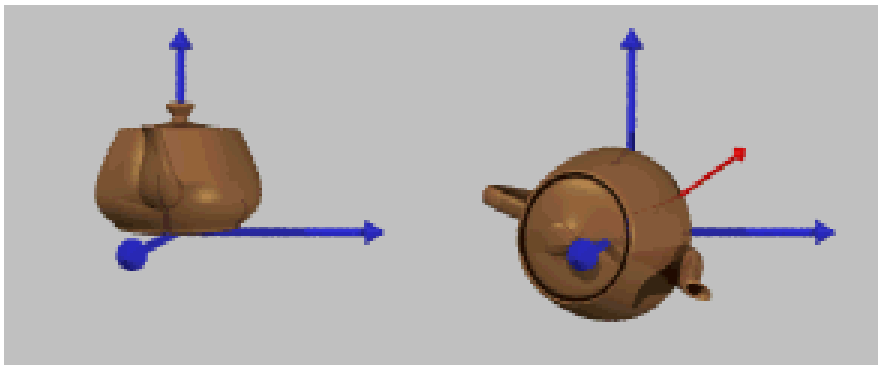


Rigid boby transformation

- Bao gom phep tònh tien va phep quay va cac to hôp cua chung.
- Do khong lam thay ñoi hình dang va kích thöôc ñoi töông, chæ lam thay ñoi vò trí, phöông höông cua chung trong khong gian.



Ví dui ve phep tònh tien



Ví dui ve phep quay